Non Linear Control: Geometric Methods and Applications

Program and Abstracts

Florence, April 18th-19th 2013

Program

Thursday 18/04		Friday 19/04	
9:00–9:30	Registration and Opening	9:05–9:50	Papini
9:30–10:15	Altafini	9:50–10:25	Chisci
10:15–10:50	Bacciotti	10:25–10:50	Rodrigues
10:50–11:20	Coffee Break	10:50–11:20	Coffee Break
11:20–12:05	Poggiolini	11:20–12:05	Sarychev
12:05–12:40	Ancona	12:05–12:40	Rampazzo
12:40–14:30	Lunch	12:40–14:30	Lunch
14:30 -15:15	Boscain	14:30 -15:15	Barilari
15:15–15:50	Rifford	15:15–15:50	Mugnai
15:50–16:20	Coffee Break	15:50–16:20	Coffee Break
16:20–16:45	Chittaro	16:20–16:45	Prandi
16:45–17:10	Fragnelli	16:45–17:10	Rizzi
17:10–18:10	Discussions	17:10–17:35	Rossi
20:00	Social Dinner		

Abstracts

Dynamics and Control on Networks with Antagonistic Interactions

Claudio Altafini

In certain contexts, quite common for example in social network theory, the interaction between the agents can be of collaborative nature but also of antagonistic nature. In the graph that represents the network of interactions, the former are described by edges with positive weights, and the latter by edges with negative weights. In this talk I will discuss what kind of dynamics can be associated with such signed graphs, in particular how a widely used property of social networks, called structural balance, induces dynamics that are monotone. When a structurally balanced signed graph is intended as a distributed control system, it is possible to design feedback laws that reach a form of bipartite consensus (i.e., a common value for all agents, up to the sign), provided that the Laplacian is defined appropriately.

Patchy Feedbacks for Stabilization and Optimal Control

Fabio Ancona

A class of discontinuous vector fields, called "patchy vector fields", that were introduced in [F. Ancona, A. Bressan, Patchy vector fields and asymptotic stabilization, ESAIM COCV 4 (1999)] to study feedback stabilization problems, revealed to be quite appropriate also for addressing the problem of constructing optimal (or nearly-optimal) feedback laws. After recalling the basic definitions and properties of patchy vector fields we shall discuss applications to control problems that has been pursued in the last years.

On some controllability notions for families of linear vector fields

Andrea Bacciotti

Global controllability of families of linear vector fields was successfully studied in the 80's using the methods of Lie groups and Lie Algebras. In this talk, we consider in addition asymptotic controllability, radial controllability and directional controllability. Asymptotic controllability is related to the stabilization problem. Radial controllability and directional controllability are equivalent to global controllability of certain associated families of nonlinear vector fields respectively on the sphere and on the projective space. We study some relationships among these forms of controllability.

The curvature: a variational approach

Davide Barilari

In this talk I will present a notion of curvature for an affine control system that is related with the asymptotic of the second derivative of the cost along a minimizing trajectory.

This notion is naturally invariant by state and feedback transformations preserving the cost.

Moreover I will discuss some application of this for drift-less systems with quadratic costs, i.e. control systems associated with the geodesic problem in (sub-)Riemannian geometry. In particular we will discuss an asymptotic formula for the sub-Laplacian of the distance squared along a geodesic.

This is a part of a joint project with A. Agrachev, L. Rizzi and P. Lee.

A spectral condition for the controllability of finite dimensional quantum systems

Ugo Boscain

For a n-level finite-dimensional closed quantum system driven by two external fields, we provide a sufficient condition for exact controllability over the sphere, based only upon spectral properties of the Hamiltonian as a function of the two controls.

This result is proven by using two ingredients that are both interesting by themselves: 1. certain purely spectral properties imply approximate controllability; 2. approximate controllability implies exact controllability for the class of systems under considerations. The second result holds in the much more general setting where the system depends nonlinearly on an arbitrary number of controls.

Observability decompositions for nonlinear sensor networks

Luigi Chisci

In state estimation problems involving wireless sensor networks, the need often arises to design a local estimator for a nonlinear unobservable system. In such a situation, it is clearly impossible to construct a stable estimator for the whole state of the system. Nevertheless it may still be possible to find a suitable observability decomposition, i.e. a possibly nonlinear mapping that splits the transformed state into observable and unobservable components, and then design a stable filter operating in the observable state space. This talk presents observability decompositions useful for specific "distributed target tracking" problems of practical interest (i.e. estimation of position and velocity of a moving target given range-only or angle-only or Doppler-only measurements provided by multiple non-colocated sensors).

Minimum-time strong optimality of a singular arc: the multi-input non-involutive case

Francesca Carlotta Chittaro

We consider the minimum time problem for a multi-input control-affine system. We use Hamiltonian methods to prove that the coercivity of a suitable second variation associated to a Pontryagin singular arc is sufficient to prove its strong-local optimality.

New results for degenerate parabolic equations

Genni Fragnelli

We present some results about well-posedness and controllability for degenerate parabolic equations where the degeneracy occurs in the interior of the domain.

A nonlinear self-obstacle control problem

Dimitri Mugnai

We consider an optimal control problem for a nonlinear elliptic variational inequality where the obstacle is the control itself. Detailed characterizations of the optimal solution are given.

Non-trivial, non-negative periodic solutions of a system of singular-degenerate parabolic equations with nonlocal terms

Duccio Papini

We study the existence of non-trivial, non-negative periodic solutions for systems of singular-degenerate parabolic equations with nonlocal terms and satisfying Dirichlet boundary conditions. The method employed in this paper is based on the Leray-Schauder topological degree theory. However, verifying the conditions under which such a theory applies is more involved due to the presence of the singularity. The system can be regarded as a possible model of the interactions of two biological species sharing the same isolated territory, and our results give conditions that ensure the coexistence of the two species.

On optimality and structural stability of extremals having both bang and singular arcs

Laura Poggiolini

We review some recent results on second order sufficient conditions for strong local optimality and structural stability of Pontryagin extremals in the minimum time problem when such extremals are concatenations of both bang and singular arcs.

Some open problems and research perspectives are presented.

The Laplace-Beltrami operator on conic-type surfaces

Dario Prandi

In this talk we consider the heat flow induced by the (generalized) Riemannian metric on $M = \mathbb{R} \times \mathbb{S}^1$, defined as

$$g_\alpha = dx^2 + |x|^{-2\alpha} dy^2, \qquad \alpha \in \mathbb{R}.$$

This metric can be interpreted geometrically as a cone of revolution with profile $x \mapsto x^{-\alpha}$, if $\alpha \leq -1$, regularly imbedded in $\mathbb{R}^3$. For $\alpha > -1$ the embedding is singular (except for $\alpha = 0$ that corresponds to the flat cylinder), and $\alpha = 1$ corresponds to the Grushin metric on the cylinder. In particular we are interested in the stochastic completeness problem (i.e. if the Markov process associated with the Laplace-Beltrami operator Δ_α has almost surely an infinite lifespan).

In order for the heat flow $\{e^{-t\Delta_\alpha}\}_{t\geq 0}$ to be well defined, Δ_α has to be a self-adjoint operator on $L^2(M)$. We will show that for $\alpha \notin (-3, 1)$, Δ_α is essentially self-adjoint, and thus there exists only one possible dynamic, that splits the two sides of the singularity. On the other hand, for $\alpha \in (-3, 1)$, we will describe the 3-parameters family of possible self-adjoint extensions, some of which will induce mixing dynamics.

In the last part of the talk, we will discuss when $\{e^{-t\Delta_\alpha}\}_{t\geq 0}$ defines a Markov process, and when these Markov processes are stochastically complete. This will be achieved through the theory of Dirichlet forms.

This is a joint work with Ugo Boscain.

Inverted pendulums, rolling balls, and other toys

Franco Rampazzo

In the mechanical motion controlled by internal moving constraints, the Riemannian structure induced by the Kinetic Energy gives the equations a structure of quadratic polynomials in the derivatives of the control coordinates. In particular, the actual presence of quadratic terms, which is common to e.g. centrifugal forces, multiple (Kapitza) pendulums, and gyroscopes, may be interpreted as the manifestation of a certain curvature term. On the other hand, when non-holonomic constraints are acting on the whole system, the quadratic terms may have a different origin, like in the case of a ball rolling on a rotating platform.

Geometric control in dynamics

Ludovic Rifford

We survey recent results in dynamics which involve techniques from geometric control theory.

Asymptotic measure contraction property in sub-Riemannian geometry

Luca Rizzi

In this short talk, we discuss homotheties in sub-Riemannian geometry and we study how the volume of measurable sets shrinks when the set collapses toward the center of the homothety. In this setting we define the *geodesic dimension* as the integer N such that the volume shrinks to zero as t^N for $t \to 0$. We show that the definition does not depend on the choice of the measure, thus it is an invariant of the sub-Riemannian structure. Under some genericity assumptions, and by exploiting the close connection with the theory of Jacobi curves, we compute the geodesic dimension. In the Riemannian case N is equal to the topological dimension of the manifold while, in the sub-Riemannian case, it is strictly greater, and also greater than the Hausdorff dimension. Finally, we briefly discuss how the geodesic dimension might be related with the classical measure contraction property of sub-Riemannian manifolds.

Anthropomorphic image reconstruction based on optimal control

Francesco Rossi

In this talk we provide an algorithm of image reconstruction based on a mathematical model of human perception initially due to Petitot, then refined by Citti and Sarti, and Agrachev and collaborators.

Given an image, considered as its gray-level function on the plane, the model is based on two crucial assumptions.

1. The visual cortex V1 takes into account the direction of the signal at each point of the plain.

Neurons of V1 are grouped into *orientation columns*, each of them being sensitive to visual stimuli in a given point a of the retina and a given direction p on it. The retina is modeled by the real plane, hence $a \in \mathbb{R}^2$, while the directions in a given point are modeled by the projective line, i.e. $p \in P^1$. Hence, the visual cortex V1 is modeled by the tangent bundle $PT\mathbb{R}^2 := \mathbb{R}^2 \times P^1$.

2. If the image is corrupted or missed on a set $\Omega \subset \mathbb{R}^2$ (i.e. if f is defined on $\mathbb{R}^2 \setminus \Omega$), then **the reconstruction of f in Ω is made by minimizing a cost representing the energy that the visual cortex should spend to excite orientation columns corresponding to points in Ω,** that are not directly excited by the image. An orientation column is easily excited if it is close to another (already activated) orientation column sensitive to a similar direction in a close position.

This minimization is described by an hypoelliptic heat equation naturally associated to the minimization problem on $PT\mathbb{R}^2$.

The algorithm presented in this paper consists in implementing the two ideas presented above. Simulations of image reconstruction performed with this algorithm will be presented.

On the local exact boundary controllability of 3D Navier–Stokes equations

Sergio S. Rodrigues

Given an arbitrary open subset of the boundary, the exact controllability to trajectories of the Navier–Stokes equations, by means of boundary controls supported in that subset, is proven. Null controllability of the Oseen system is investigated as well, and continuity results of the driving control on the initial data are derived.

Geometric Control Methods for Equations of Fluid Motion

Andrey Sarychev

We review part of the research activity (joint with A.Agrachev and S.Rodrigues) on controllability issues for the 2D Euler and Navier-Stokes systems, which describe the evolution of the velocity field of a homogeneous ideal or viscous incompressible fluid on a two-dimensional domain. The control, applied via forcing term, is low-dimensional.

We treat an extended problem setting, which regards steering the velocity field and a domain of fluid to target velocity field and domain respectively. Sufficient approximate controllability criterion is presented.