Asymptotic ensemble stabilizability of the Bloch equation

F. C. Chittaro and J.-P. Gauthier

The notion of *ensemble controllability* (also called *simultaneous controllability*) concerns the controllability of parametrized families (ensembles) of control systems, from some initial state to some prescribed target state, with the same control for the whole family. This issue is motivated by recent engineering applications, such as, for instance, quantum control and distributed parameters systems.

In this talk, we are concerned with the simultaneous control of an ensemble of spin immersed in a magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$; in this model, each spin is described by the magnetization vector $\boldsymbol{M} \in \mathbb{R}^3$, subject to the dynamics $\frac{\partial M}{\partial t} = \boldsymbol{B} \times \boldsymbol{M}$ (Bloch equation).

We consider the case in which the z-component of the magnetic field is constant and non-uniform with respect to the position of the spins, while the other two components are uniform and can be controlled. Setting $B_y(t) = u_1(t)$ and $B_x(t) = -u_2(t)$, the dynamics become

$$\frac{\partial \boldsymbol{M}}{\partial t}(\boldsymbol{r}, t) = \begin{pmatrix} 0 & -B_z(\boldsymbol{r}) & u_1(t) \\ B_z(\boldsymbol{r}, t) & 0 & u_2(t) \\ -u_1(t) & -u_2(t) & 0 \end{pmatrix} \boldsymbol{M}(\boldsymbol{r}, t). \tag{1}$$

Coupling a Lyapunov function approach with some tools of dynamical systems theory, we exhibit a control function (in feedback form) that approximately drives, asymptotically in time and generically with respect to the initial conditions, all spins to the "down" position $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$. Two cases are addressed: if the set of spins is finite, our strategy provides exact exponential stabilizability in infinite time, while if we have a countable collection of spins, our approach implies asymptotic pointwise convergence towards the target state.